Calculus Warm Up #3-5

$$\frac{dy}{dx} = \frac{3 - x}{y}$$

Let $y = g(x)$ be the particular solution to yesterday's differential equation for $-2 < x < 8$, with initial condition $g(6) = -4$. Find $y = g(x)$.

Turn in:

<u>Week 3 Classwork</u>

Warm up on top

Green FR 2010

HW Questions: p. 591

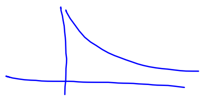
7) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$ compare to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n+1}$

$$0 \le \frac{1}{n+1} \le \frac{\ln n}{n+1}$$

Integral Test

$f(x) = \frac{1}{x+1}$

positive ✓
continuous ✓
decreasing ✓



$$\lim_{b\to\infty} \int_2^b \frac{1}{x+1}\,dx = \lim_{b\to\infty} \Big[\ln(x+1)\Big]_2^b$$
$$= \lim_{b\to\infty} \big(\ln(b+1) - \ln 2\big)$$
$$= \infty$$

9) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$ compare to $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2}$

n	1	2	3	4
$\frac{1}{n^2}$	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$
$\frac{1}{n!}$	1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{24}$

(smaller for $n > 3$

$$0 \le \frac{1}{n!} \le \frac{1}{n^2}$$

Since $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2}$ p-series; $p = 2 > 1$
converges for $n > 3$

$\therefore \displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$ also converges.

11) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$ Compare to $\displaystyle\sum_{n=0}^{\infty} \frac{1}{e^n}$ Geometric $r = \frac{1}{e} < 1$ converges

$\frac{1}{e^n}$

$\frac{1}{e^{n^2}}$

$\dots \quad 0 \le \dfrac{1}{e^{n^2}} \le \dfrac{1}{e^n}$

$\therefore \displaystyle\sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$ converges.

15) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ compare to $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2}} \to \frac{1}{n}$ divergent p-series

let $a_n = \dfrac{1}{\sqrt{n^2+1}}$

$b_n = \dfrac{1}{n}$

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = ?$

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = \boxed{1}$$

positive & finite

Both series same

$\boxed{\therefore \displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}} \text{ Diverges}}$

17) compare to $\sum \frac{n^2}{n^5} \to \sum \frac{1}{n^3}$

19) $\sum_{n=1}^{\infty} \frac{n+3}{n(n+2)}$

$\downarrow$

$\frac{n+3}{n^2+2n}$

compare to

$\sum \frac{n}{n^2} \to \frac{1}{n}$

21) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$ compare to $\sum \frac{1}{n\sqrt{n^2}}$

$\downarrow$

$\frac{1}{n^2}$

$p = 2 > 1$ converges

$$\lim_{n\to\infty} \frac{\frac{1}{n\sqrt{n^2+1}}}{\frac{1}{n^2}} \cdot \frac{n^2}{1}$$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = 1$$

23) $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+1}$, $k > 2$

compare to

$\sum \frac{n^{k-1}}{n^k} \to \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{n^{k-1}}{n^k+1}}{\frac{1}{n}} \cdot \frac{n^1}{1}$$

$$\lim_{n\to\infty} \left[\frac{n^k}{n^k+1}\right] = 1$$

Classwork: Continue with

2002 FR # 1 - 3 (Purple)

A: intersection $x \approx 1.488$ stored.

1a) $A = \int_0^A \left[4 - 2x - \left(\frac{x^3}{1+x^2}\right)\right] dx$

$A \approx 3.215$

b) $R = 4 - 2x$

$r = \frac{x^3}{1+x^2}$

$V = \pi \int_0^A \left[(4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right] dx$

$V \approx 31.885$

c) $V = \int_0^A \left(4 - 2x - \left(\frac{x^3}{1+x^2}\right)\right)^2 dx \approx 8.997$

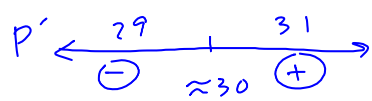
2a) increasing if $P'(9) = +$

$P'(9) \approx -0.646$, so amount of P is decreasing at $t = 9$ days

b) min where $P'(t) = 0$ & $P'(t)$ goes from − to +

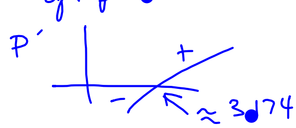
$0 = 1 - 3e^{-0.2\sqrt{t}}$

$t \approx 30.174$ days



confirms min at $t \approx 30.1$ days

* could also graph:



c) Safe if $P(\approx 30.1) \leq 40$

$P(\approx 30.1) = 50 + \int_0^{\approx 30.1} P'(t)\,dt$

≈ 35.104

Since $35.104 < 40$, yes the lake is safe when P is at its minimum.

2d) tangent line: $y - 50 = -2(t - 0)$

$y = -2t + 50$

Safe when $y \le 40$

So: $-2t + 50 \le 40$

$t \ge 5$

linear model predicts the lake becomes safe when $t \ge 5$ days.

HW: Pink, FR 2008 #3

No HW Quiz next week

We will continue with CH 10

and AP FR practice.